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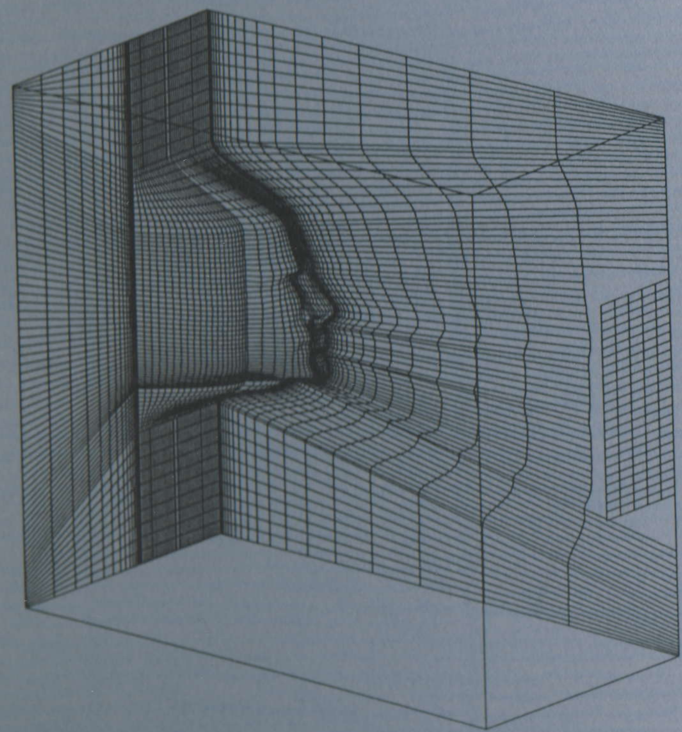

DYNAMIC ANALYSIS OF STRUCTURES WITH UNILATERAL CONSTRAINTS: NUMERICAL INTEGRATION AND REDUCTION OF STRUCTURAL EQUATIONS

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Direct integration of structural equations with unilateral constraints. Consider the structural equation of motion

$$M \ddot{U} + C \dot{U} + K U = R(t) \quad , \tag{1}$$

with the constraints upon the displacements

$$P U \le d_o \quad , \tag{2}$$

and, when they are satisfied as an equality $PU-d_o=0$, the auxiliary constraints upon the m time derivatives of the displacements U are imposed as

$$P \overset{(k)}{U} = d_k \quad , \quad k = \overline{1,m} \quad . \tag{3}$$

where M, C, K of dimension $n{\times}n$ and P of dimension $p{\times}n$, $p{\le}n$, are the constant or time-dependent matrices, and U, R of dimension $n{\times}1$ - the vectors of nodal displacements and external forces. The physical meaning of the constraints (2) is the non-penetration condition of the contacting surfaces into each other in the case when the pairs of possible contact points are known apriori. The constraints (3) enable to represent the local impact condition between the contacting points in terms of the impact restitution coefficient.

Employing the Lagrange multipliers we obtain the system

$$\begin{cases} M \ddot{U} + C \dot{U} + K U + P^T \lambda_o = R \ , \\ P U \le d_o \ , \\ P \overset{(k)}{U} = d_k \ , \quad k = \overline{1,m} \ , \end{cases} \tag{4}$$

where only nonnegative values of λ_o are allowed, i.e., each j-th component of the Lagrange multiplier vector λ_o is defined as

$$\lambda_{oj} = \begin{cases} \lambda_{oj}, & \text{if } \lambda_{oj} \geq 0, \\ 0, & \text{otherwise} \end{cases} .$$

and integrate it numerically applying a single step scheme. If at the time point $t+\Delta t$ the second relation of (4) is satisfied as an equality and the third one can't be satisfied for all or some values of k, the values of velocities, accelerations and higher time derivatives $\overset{(k)}{U}$ must be corrected.

Assuming that the corrections of the velocities are carried out during a very short time interval Δt_s in comparison with the duration of the integration step and employing the Carnot's theorem we require that the variation of the kinetic energy of the system because of introducing a new constraint should be equal to the kinetic energy of lost velocities $\frac{1}{2}\Delta\dot{U}^T M \Delta\dot{U}$. This loss of energy is caused by the work done by the contact forces during the time interval Δt_s. The motion of the system corresponds to the minimum value of this work, or, what is the same, to the minimum change of kinetic energy. At the end of the interval $(t, t+\Delta t_s)$ the constraint upon the velocities of the system must be satisfied, i.e., it is necessary to solve the system

$$\begin{cases} \min \frac{1}{2}\Delta\dot{U}^T M \Delta\dot{U}, \\ \text{with the constraint } P\Delta\dot{U} = -P\dot{U}^- + d_1 \end{cases},$$

obtaining

$$\lambda_1 = (P M^{-1} P^T)^{-1}(P\dot{U}^- - d_1),$$

$$\Delta\dot{U} = -M^{-1}P^T(P M^{-1}P^T)^{-1}(P\dot{U}^- - d_1) .$$

Similarly we obtain all λ_k and $\Delta\overset{(k)}{U}$, $k \leq m$ for the arbitrary derivative order m. The physical meaning of the Lagrange multipliers λ_1, λ_2, λ_3,... is the normal impetus, forces, derivatives of forces, etc., representing the action of the constraints upon the structure during the time interval $(t, t+\Delta t_s)$.

The impetus of the constraint forces in global coordinates during the time interval $(t, t+\Delta t_s)$ is defined as

$$S_1 = - P^T S_N = - P^T (P\, M^{-1} P^T)^{-1} (P\, \dot{U}^- - d_1) \quad .$$

The forces of the constraints enabling to change the acceleration value during the time interval $(t, t+\Delta t_s)$ are defined as

$$F_1 = - P^T F_N = - P^T (P\, M^{-1} P^T)^{-1} (P\, \ddot{U}^- - d_2) \quad .$$

The numerical integration scheme has been obtained employing as a basis the generalized Newmark's scheme. The impetus of the normal interaction forces S_N during the time interval equal the integration step Δt are approximately obtained as

$$S_N^{t+\Delta t} = \frac{F_N^t + \lambda_c^{t+\Delta t}}{2} \Delta t + \lambda_1^{t+\Delta t} \quad .$$

The normal forces F_N ensuring the dynamic equilibrium at the time point $t+\Delta t$, are obtained as

$$F_N^{t+\Delta t} = \lambda_o^{t+\Delta t} + \lambda_2^{t+\Delta t} \quad .$$

Reduction of structural equations with unilateral constraints.

The approach presented below is carried out by truncating the dynamic contributions of the higher modes of the linear part of the structure, simultaneously taking into account the remaining structural compliance of the dynamically truncated modes.

Consider the structural equation of motion (1) with a proportional damping $C = \alpha M + \beta K$. By solving the eigenproblem

$$(K - \omega^2 M)\, U = 0 \quad ,$$

there are obtained the eigenfrequencies ω_i, $i=\overline{1,n}$ and the eigenvectors, that are ordered as columns of the matrix Y. The presentation in modal coordinates is carried out by means of the substitution

$$U = Y z ,$$

where Z – the generalized (modal) displacements of the structure.

We present the vector of squares of eigenfrequencies as $\begin{bmatrix} \omega_1^2 \\ \omega_2^2 \end{bmatrix}$, and the matrix of eigenvectors as $Y = [Y_1, Y_2]$, where the submatrix Y_2 and the subvector ω_2^2 correspond to the modes subjected to the dynamic truncation. After the truncation, the equation (1) with the constraints (2) in modal coordinates takes the form

$$\begin{cases} I \ddot{z}_1 + \mathrm{diag}(\mu_1)\, \dot{z}_1 + \mathrm{diag}(\omega_1^2)\, z_1 = Y_1^T (R - P^T \lambda) , \\ \mathrm{diag}(\omega_2^2)\, z_2 = Y_2^T (R - P^T \lambda) \\ P\, Y_1 z_1 + P\, Y_2 z_2 = d_o \end{cases} \qquad (5)$$

where $\mathrm{diag}(\mu_1)$, $\mathrm{diag}(\omega_1^2)$, $\mathrm{diag}(\omega_2^2)$ denote the diagonal matrices containing the vectors μ_1, ω_1^2, and ω_2^2 in their main diagonals, and the relations $\mathrm{diag}(\mu_1) = Y_1^T C\, Y_1$, $\mathrm{diag}(\omega_1^2) = Y_1^T K\, Y_1$, $\mathrm{diag}(\omega_2^2) = Y_2^T K\, Y_2$ are held.

The system (5) is a reduced one in comparison with the original equation. The values of the Lagrange multipliers λ denote normal interaction forces produced by constraints upon a structure. They are obtained as

$$\lambda(z_1) = (P\, S_k P^T)^{-1} (P\, Y_1 z_1 + P\, S_k R - d_o) ,$$

where

$$S_k = Y_2 \mathrm{diag}(1/\omega_2^2) Y_2^T = K^{-1} - Y_1 \mathrm{diag}(1/\omega_1^2) Y_1^T ,$$

only nonnegative values $\lambda_j \geq 0$ being allowed.

The numerical examples illustrate the presented techniques by considering free longitudinal impact vibrations of a beam vibroconverter employing full and reduced structural models, and a free motion of a vibrodrive.